INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Second Year, Second Semester, 2011-12
Statistics - II, Midterm Examination, February 24, 2012

1. Suppose $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ are independent random samples, respectively, from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, where $-\infty < \mu_1, \mu_2 < \infty$, $\sigma_1^2 > 0$, $\sigma_2^2 = 2\sigma_1^2$.
(a) Does this model belong to the exponential family of distributions? Justify.
(b) Find minimal sufficient statistics for the unknown parameters. Is it complete?
(c) Find the MLE and UMVUE of σ_1^2. [15]

2. Consider a trial which ends up in 'Success' with probability p or 'Failure' with probability $1 - p$, $0 < p < 1$. Let X denote the number of independent trials required to obtain the first 'Success'.
(a) Find the probability mass function of X.
(b) Find the Fisher information of p contained in X.
(c) Let $X_1, \ldots, X_n$ be a random sample from the distribution of X. Find the Cramer-Rao lower bound on the variance of an unbiased estimator of $\frac{1}{p}$ based on this random sample. [10]

3. Consider a random sample $X_1, X_2, \ldots, X_n$ from any distribution with a finite variance $\sigma^2 > 0$.
(a) Show that $\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is an unbiased estimator of σ^2.
Now suppose that X_i are i.i.d. Bernoulli(p), $0 < p < 1$.
(b) Find the complete sufficient statistic for p.
(c) Show that $\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is the UMVUE of p. [12]

4. Consider a family of regular models with density $f(x|\theta)$ such that $f(x|\theta) > 0$ for all $\theta \in \Theta$ and for all $x \in \mathcal{X}$. Suppose $T(X)$ is sufficient for this family of distributions indexed by θ. Show that if $T(x) = T(y)$ for two sample points x and y then $\frac{f(x|\theta)}{f(y|\theta)}$ is free of θ. [5]

5. Consider a family of regular models with density $f(x|\theta)$ such that $f(x|\theta) > 0$ for all $\theta \in \Theta$ and for all $x \in \mathcal{X}$. Suppose $\Theta \subseteq \mathcal{R}^k$ where $k \geq 1$. A random sample, $X_1, X_2, \ldots, X_n$ of size $n > k$ is available. In this set-up give an example each of the following kind.
(a) Method of moments estimator of θ exists but MLE does not exist.
(b) MLE of θ exists but Method of moments estimator does not exist. [8]